Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name: Complex Analysis

Subject Code: 5SC01COA1 **Branch: M.Sc.(Mathematics)**

Semester: 1 **Date:30/11/2018** **Time:02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a) Find the rectangular form of $z = 4e^{-570^\circ i}$. (02)

b) Solve: $\sin z = 2$. (02)

c) Define Entire function and give one example of it. (02)

d) True/False: An analytic function with constant imaginary part is constant. (01)

Q-2 **Attempt all questions** **(14)**

a) If $\alpha, \alpha^2, \alpha^3, \alpha^4$ are the roots of $x^5 = 1$, find them and prove that $(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4) = 5$. (04)

b) Show that the set of values of $Log\, i^3$ is not same as the set of values of $3Log\, i$. (03)

c) Find real and imaginary part of $\left(\sqrt{i}\right)^{\sqrt{i}}$. (04)

d) Find the value of $\tan^{-1}(2i)$. (03)

OR

Q-2 **Attempt all questions** **(14)**

a) Find the product of all roots of $z^5 = 1 + i$. (04)

b) Solve: $z^2 - (3-i)z + (4-3i) = 0$ (03)

c) Find real and imaginary part of $\left[\frac{e}{2}\left(-1-\sqrt{3}i\right)\right]^{3\pi i}$. (04)

d) Prove that $\sinh^{-1} z = \log\left(z + \sqrt{z^2+1}\right)$. (03)

Q-3 **Attempt all questions** **(14)**

a) State and prove necessary and sufficient condition for the function to be analytic. (07)

b) Find the analytic function $f(z)$ if the real part of $f(z)$ is $r^2\cos 2\theta + r\sin\theta$. (05)

c) Define: Harmonic function, Differentiability of function. (02)

OR

Q-3 **Attempt all questions** **(14)**

a) State and prove chain rule for derivatives. (07)

b) If $f(z)=\sqrt{|xy|}$ is not analytic at the origin although Cauchy-Riemann equations are satisfied at origin. (05)

c) Define: Continuous function and give one example of the function which is continuous but not differentiable. (02)

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a) State Liouville's theorem. (02)

b) Evaluate: $\oint_C \frac{e^z}{z(z-1)^3}dz$, c: $|z| = \frac{1}{2}$. (02)

c) Find the residue of $f(z)=z^4 e^{\frac{1}{z}}$. (02)

d) Which are the fixed points of $w=\frac{5z-4}{5+z}$? (01)

Q-5 **Attempt all questions** **(14)**

a) State and prove Cauchy's integral formula. (05)

b) Integrate the function $f(z)=(\bar{z})^2$ from 0 to $2+i$ path is from $(0,0)$ to $(2,0)$ along the real axis and then from $(2,0)$ to $(2,1)$. (05)

c) Integrate the function $f(z)=\frac{1}{z^4+4z^2}$ around the curve $C:|z-2i|=3$ traversed in counter-clockwise direction. (04)

OR

Q-5 **Attempt all questions** **(14)**

a) Evaluate $\oint_C \frac{\sin \pi z^2+\cos \pi z^2}{(z-2)(z-1)^2}dz$; $C:|z|=3$ by using Cauchy's integral formula. (05)

b) Let $P(z)=a_0+a_1z+a_2z^2+....+a_nz^n\ (a_n\neq 0)$ be a complex valued polynomial of degree $n(n\geq 1)$ then there exist at least one complex root z_0 such that $P(z_0)=0$. (05)

c) Find an upper bound for the absolute value of the integral $\int_C \frac{\sqrt{z}}{z^2+1}dz$ where c is the contour given by upper half of the circle $|z|=3$. (04)

Q-6 Attempt all questions (14)

a) Find the Laurent expansions for the function $f(z)=\dfrac{z}{(z-2)(z+i)}$ in the regions (05)

i) $1<|z|<2$ ii) $|z|>2$.

b) Evaluate: $\displaystyle\int_0^{2\pi}\frac{\cos\theta}{5+4\cos\theta}\,d\theta$ (05)

c) Find the bilinear transformation which maps $1, i, -1$ onto $i, 0, -i$ respectively and also find the image of $|z|<1$. (04)

OR

Q-6 Attempt all Questions (14)

a) State and prove Taylor's theorem. (05)

b) Evaluate: $\displaystyle\int_{-\infty}^{\infty}\frac{dx}{(x^2+1)(x^2+2x+2)}$ (05)

c) Evaluate $\displaystyle\oint_C\frac{z}{(z-2)^2(z-1)}\,dz;\ c: |z-2|=0.5$ by using Cauchy's residue theorem. (04)